Curve Sketching.

Even function:- If $f(-x) = f(x)$
Eg:- $\cos x$, x^2, etc.
↓
$\boxed{\text{Symmetric about y-axis}}$

Odd function:- If $f(-x) = -f(x)$
Eg:- $x^3 + x$, $\tan x$, etc.
↓
$\boxed{\text{symmetric about origin}}$

Periodicity: Repeating of graph pattern in certain interval

To test periodicity,
① Find period (k) by
$$k = \frac{\alpha}{|a|}$$
$\alpha \rightarrow$ Fundamental period
$a \rightarrow$ constant term of angle
② Show, $f(x) = f(x+k)$

Fundamental period : (i) Of $\sin x$ & $\cos x$ is 2π
(ii) Of $\tan x$ is π.

Increasing function: A function $f(x)$ is said to be increasing in an interval (a,b) if for all $x_1, x_2 \in (a,b)$
$$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$$

Decreasing function: A function $f(x)$ is said to be decreasing in an interval (a,b) if for all $x_1, x_2 \in (a,b)$
$$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$$

$\boxed{x_1, x_2 \in \text{Given interval}}$

Characteristics for sketching the curve of the function:-

(i) Origin [Passes or not] [By keeping $x=0$, if $y=0$; passes]
(ii) Domain & Range [x र y-axes मा कति space लिन्छ]
(iii) Symmetricity [Even function - Symmetric about y-axis
Odd function - Symmetric about origin]
(iv) Periodicity [For trigonometric function]
(v) Asymptote: [For Logarithmic, exponential, tangent]
↳ [If $x = a$, $y \rightarrow \infty$ (vertical asymptote)
If $y = a$, $x \rightarrow \infty$ (horizontal asymptote)]
(vi) Increasing and decreasing region.
(vii) Openness [quadratic, $ax^2+bx+c=0$ $a<0$ opens downward
$a>0$ opens upward]
(viii) Vertex [For quadratic, $(h,k) = \left(\frac{-b}{2a}, \frac{4ac-b^2}{4a}\right)$]
(ix) points on axes

Quadratic function
Characteristics:-
(i) Openness ($a>0; a<0$) (ii) Origin (iii) Vertex (h,k) (iv) line of symmetry $\boxed{x=h}$

(v) cutting x-axis (at $y=0$) (vi) Domain $=(-\infty,\infty)$
Range $=[k,\infty)$ if $a>0$
$[-\infty,k]$ if $a<0$.

For cubic function:-
(i) Origin (ii) Symmetricity (iii) Increasing and decreasing parts.

(iv) Point on axes. (v) Domain $=(-\infty,\infty)$
Range $=(-\infty,\infty)$

For sine and cosine function:-
(i) Origin (ii) Amplitude [$y=b\sin ax$ or $y=b\cos ax$
$b\rightarrow$ amplitude]

(iii) Point on axes (iv) Period $(K)=\frac{\alpha}{|a|}$ (v) Domain $(-\infty,\infty)$ or given interval
Range $[-b,b]$

(vi) Symmetricity [Odd $\rightarrow$ origin Even $\rightarrow$ y-axis]

For tangent function [$y=a\tan bx$]
(i) Origin (ii) Domain $=(-\infty,\infty)$ or given interval; Range $=(-\infty,\infty)$

(iii) Period $(K)=\frac{\alpha}{|a|}$ (iv) Symmetricity (origin)

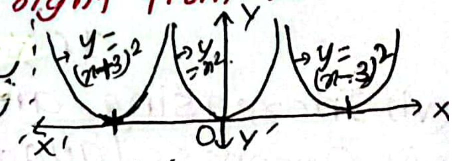
(v) Points on axes (vi) Asymptote.

For exponential function [$y=\log_a x \Rightarrow x=a^y$]
(i) Origin (ii) Asymptote (iii) Points where A +ve / A −ve $\downarrow$ lie above/below x/y axes.

(iv) Points on axes (v) cuts x-axis; y-axis

(vi) Increasing & decreasing.

Horizontal shifting of curve:- The graph of $f(x-a)$ is same as the graph of $f(x)$ moved to 'a' units right from the graph of $y=f(x)$. Note,



$y=f(x-a)$ $(a>0)$, shifted a units right
$y=f(x+a)$, $(a>0)$, shifted a units left

Vertical shifting of curve:-
(i) Graph of $y=f(x)+b$ $(b>0)$ is graph of $y=f(x)$ shifted b unit upward.

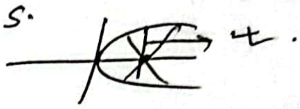
(ii) Graph of $y=f(x)-b$ $(b>0)$ is graph of $y=f(x)$ shifted b units downward.

Focal distance $= x + a$ } For horizontal parabola
Focal distance $= y + a$ } For vertical parabola,

For vertex → Partial derivative (w.r.t x & w.r.t y)
Axis → Partial derivative

- Length of Latus rectum ⇒ $y^2 \Rightarrow x$ ko coeff. { keeping coeff. of y^2 1 }
  ⇒ $x^2 \Rightarrow y$ ko coeff.

- Vertex is shortest distance from focus.
- Latus rectum is shortest focal chord



- If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are extremities of focal chord then $\boxed{t_1 t_2 = -1}$
  & If these point subtend Rt. angle at vertex then, $\boxed{t_1 t_2 = -4}$

- Locus of mid-point of chord to $y^2 = 4ax$ passing through vertex → $y^2 = 2ax$ ← ½

- $y^2 = x$ ⇒ even in y → symmetrical about x-axis
  ⇒ even in x → y-axis
  ⇒ even in both → both axes.
  { circle, ellipse, hyperbola }

Tan^{-1} $\sqrt{S_1}$ / Directrix

Tangent at vertex, linear term = 0.

- For tangent, $x^2 = 4ay$
  ⇓
  Take line to this form → $x = my + c$
  ↳ $c = a/m$
  } Interchanging method.

- Parabola → Quadratic eq^n not linear.

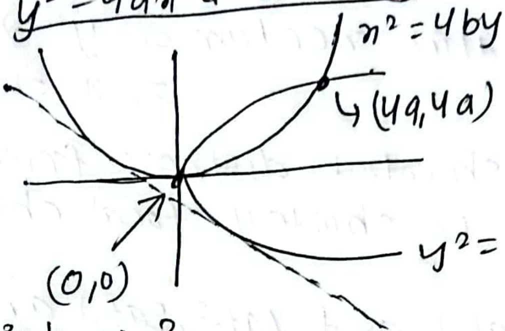
- Slope $= \frac{dy}{dx}$ } of tangent

  slope $= \frac{-1}{dy/dx}$ } Normal

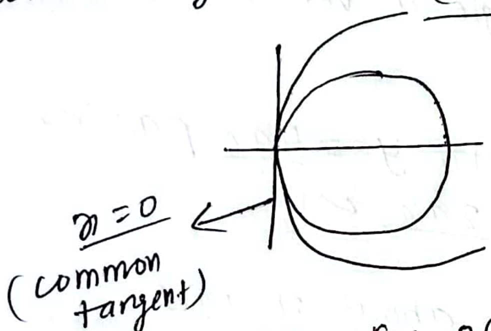
Angle b/w tangent from (x_1, y_1) to $y^2 = 4ax$
↳ $\tan^{-1}\left(\frac{\sqrt{y_1^2 - 4ax_1}}{x_1 + a}\right)$ ← Root of S_1 ; ← Directrix.

- Angle b/w tangents at end points of focal chord $\Rightarrow 90^\circ$ {Meet at directrix}
- If tangents drawn from parabola meet at directrix then Angle b/w them is 90°
- Common tangent b/w $y^2 = 4ax$ & $x^2 = 4by$

$$a^{1/3}x + b^{1/3}y + a^{2/3}b^{2/3} = 0$$



- Max. normal from a point $\rightarrow 3$
- Common tangent b/w $(x-2a)^2 + y^2 = (2a)^2$ & $y^2 = 4ax$
circle, parabola



- If normal at point $(at_1^2, 2at_1)$ meet at another point $(at_2^2, 2at_2)$ on parabola $y^2 = 4ax$, then $t_2 = -t_1 - \frac{2}{t_1}$

- Condition of tangency of line $y = mx + c$ to $y^2 = 4ax \Rightarrow c = a/m$
- Condition of tangency of $y = mx + c$ to $y^2 = 4a(\underbrace{x+a}_{})$ ↑ variable of line should be in this form

$$\Downarrow$$

$$y = m(x+a) + (c - am)$$

$$\Rightarrow (c - am) = \frac{a}{m}$$

$$\Rightarrow c = am + a/m$$

- Eq^n of normal of parabola $y^2 = 4ax$ having slope 'm'

↳ $y = mx - 2am - am^3$

& POC is $(am^2, -2am)$ } $m \rightarrow$ slope of normal.

- POC $\Rightarrow$ Trial.
- Eq^n of given parametric form } Anyhow Parameter हटाउ
- No. of tangents } Given point (x_1, y_1)
  - outside $\rightarrow 2$ } $y_1^2 - 4ax_1 > 0$
  - on $\rightarrow 1$ } $y_1^2 - 4ax_1 = 0$
  - inside $\rightarrow 0$ } $y_1^2 - 4ax_1 < 0$
- The polar of (x_1, y_1) (Pole) w.r.t. $x^2 + y^2 = a^2$ is $\boxed{xx_1 + yy_1 = a^2}$ ← Tangent को Formula lekhne at the point

https://www.surendrasharma0001.com.np/

→ $\log_e K$ ⇒ is not defined for $K \le 0$

→ No. of solutions : No of points where graphs cut each other

$f(x) = g(x)$ ⇒ curves are $y = f(x)$ & $y = g(x)$

→ ${}^nC_r = {}^nC_s \Rightarrow r = s$ or $r + s = n$

$${}^nP_r = r! \cdot {}^nC_r$$

✓ ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

✓ $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$

${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$

Selecting one or more object out of $n = 2^n - 1$.

(If only two ways for each object)

Some useful derivatives:

① $\frac{d}{dx}[f\{g(x)\}] = f'\{g(x)\}g'(x)$

② $\frac{d}{dx}x^n = nx^{n-1}$ [Eg: $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$ & $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$]

③ $\frac{d}{dx}[f(x)]^n = n\{f(x)\}^{n-1} \cdot f'(x)$

④ $\frac{d}{dx}e^{f(x)} = e^{f(x)} \cdot f'(x)$

⑤ $\frac{d}{dx}a^{f(x)} = a^{f(x)} \cdot f'(x)\log_e a$.

⑥ $\frac{d}{dx}\log f(x) = \frac{1}{f(x)} \cdot f'(x)$

⑦ $\frac{d}{dx}\log x = \frac{1}{x}$, for $x > 0$

⑧ $\frac{d}{dx}\log|x| = \frac{1}{x}$, for $x \neq 0$

⑨ $\frac{d}{dx}(\log_a x) = \frac{d}{dx}\left(\frac{\log x}{\log a}\right) = \frac{1}{x\log a}$, for $x > 0$

⑩ $\frac{d}{dx}|x| = \frac{x}{|x|}$ or $\frac{|x|}{x}$

Derivatives of trigonometric functions:

① $\frac{d}{dx}\sin x = \cos x$ ② $\frac{d}{dx}\cos x = -\sin x$ ③ $\frac{d}{dx}\tan x = \sec^2 x$

④ $\frac{d}{dx}\sec x = \sec x\tan x$ ⑤ $\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x\cot x$ ⑥ $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

Derivatives of inverse trigonometric functions:

① $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$

② $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$, $\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$

③ $\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$, $\frac{d}{dx}\operatorname{cosec}^{-1}x = \frac{-1}{x\sqrt{x^2-1}}$

Derivative of hyperbolic functions:

$\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$

1. $\frac{d}{dx} \sinh x = \cosh x$
2. $\frac{d}{dx} \cosh x = \sinh x$
3. $\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$
4. $\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$
5. $\frac{d}{dx} \operatorname{cosech} x = -\operatorname{cosech} x \coth x$
6. $\frac{d}{dx} \coth x = -\operatorname{cosech}^2 x$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

↳ provided that this limit exists and is finite

(i) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

(ii) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

(iii) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

(iv) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ } If $y = f(t)$ & $t = f(x)$

Derivative of parametric functions

If $x = \phi(t)$ and $y = \psi(t)$, then

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Partial derivative:

For $f(x,y) = 0$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

For u-substitution

$\sqrt{\boxed{u}}$, $e^{\boxed{u}}$, $\frac{1}{\boxed{u}}$

$\boxed{u}^{\boxed{x}}$ $\frac{1}{\sqrt{\boxed{u}}}$ $\frac{1}{(u)^2}$

$u) + c$

$\log \boxed{u}$

$\sin \boxed{u}$, $\cos \boxed{u}$

Formulae:-

$\log_e e^a = a$

$a^{\log_a b^n} = b^n$

Integration (Anti-Derivatives)

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad [n \neq -1]$
2. $\int e^x dx = e^x + c$
3. $\int e^{ax} dx = \frac{e^{ax}}{a} + c$
4. $\int \frac{1}{x} dx = \log x + c$
5. $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$
6. $\int \frac{1}{ax+b} \cdot dx = \frac{1}{a} \log(ax+b) + c$
7. $\int a^x dx = \frac{a^x}{\log_e a} + c$
8. $\int \sin x dx = -\cos x + c$
9. $\int \sin ax \, dx = -\frac{\cos ax}{a} + c$
10. $\int \cos x \, dx = \sin x + c$
11. ~~$\int \tan x dx = \sec^2 x + c$~~
12. $\int \sec^2 x \, dx = \tan x + c$
13. $\int \sec^2 ax \, dx = \frac{\tan ax}{a} + c$
14. $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$
15. $\int \sec x \cdot \tan x \, dx = \sec x + c$
16. $\int \operatorname{cosec} x \, \cot x \cdot dx = -\operatorname{cosec} x + c$
17. $\int \tan x \, dx = \log(\sec x) + c$
18. $\int \cot x \, dx = \log(\sin x) + c$
19. $\int \sec x \, dx = \log(\sec x + \tan x) + c$
20. $\int \operatorname{cosec} dx = \log(\operatorname{cosec} x - \cot x) + c$
21. $\int 1 \, dx = x + c$
22. $\int e^{(ax+b)} dx = \frac{1}{a} e^{(ax+b)} + c$
23. $\int k f(x) dx = k \int f(x) dx$
24. $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$
25. $\int (uv) dx = u \int v dx - \int \left(\frac{du}{dx} \int v dx \right) dx + C$, this is known as integration by parts. $u \rightarrow$ easily derivable, $v \rightarrow$ easily integrable $\hookrightarrow$ ILATE rule.
26. For, (i) $a^2 - x^2$, $x = a \sin\theta$
    (ii) $x^2 - a^2$, $x = a \sec\theta$
    (iii) $x^2 + a^2$, $x = a \tan\theta$

For u-substitution

$\sqrt{\boxed{u}}$ $\quad \boxed{e^{\boxed{u}}}$ $\quad \frac{1}{\boxed{u}}$

$\boxed{u}^{\boxed{x}}$ $\quad \frac{1}{\sqrt{\boxed{u}}}$ $\quad \frac{1}{(u)^2}$

$\log \boxed{u}$

$\sin \boxed{u}$, $\cos \boxed{u}$

Integration by parts :-

$$\int \underset{I}{u}v\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$$

$u \to 1^{st}$ funcn.
$v \to 2^{nd}$ function

$u \to$ easily derivable
$v \to$ easily integrable

Order of choosing 1^{st} f(x) →
I = Inverse funct.
L = Logarithmic f(x)
A = Algebraic fxns
T = trigo (fxn)
E = Expo. f(x).

★ If inverse function & logarithmic function has no second product then take '1' as second product.

★ If one function is x^n, $n \in N$, Then

$$\int x^n v\,dx = x^n\int v\,dx - \underline{F(x)}$$

→ $\underline{F(x)}$ is obtained by applying the rule $\frac{(\text{derivative of I})\times}{(\text{Integration of II})}$ on every previous term obtained & writing them in alternate sign.

eg: $\int x^3 e^{ax}\,dx = (x^3)\left(\frac{e^{ax}}{a}\right) - (3x^2)\left(\frac{e^{ax}}{a^2}\right) + (6x)\left(\frac{e^{ax}}{a^3}\right) - (6)\left(\frac{e^{ax}}{a^4}\right) + c$

Int. | der I

Integral of type $\int \sin^m x\cos^n x\,dx$

(i) m-odd ⇒ cos x = t
(ii) n-odd ⇒ sin x = t

जसको power odd xaina त्यसलाई 't' put garne.

Integrals of type $\int \sin mx\cos nx\,dx$, $\int \sin mx\sin nx\,dx$, $\int \cos mx\cos mx\,dx$

Use trigo identities.
(i) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
(ii) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
(iii) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
(iv) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Standard integrals of Trigo.

(a) $\int \operatorname{cosec} x\,dx = \log\left(\tan\frac{x}{2}\right) + c$

(b) $\int \sec x\,dx = \log\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) + c.$

★ Anyhow try to make $\sec^2$ in numerator and $\tan^2$ or tan in denominator.

- linear term in deno:

eg: $\int \frac{1}{a+b\sin x}dx$, $\int \frac{1}{a\sin x+b\cos x}dx$, $\int \frac{dx}{a+b\cos x}$

$\sin x = \frac{2\tan x/2}{1+\tan^2 x/2}$, $\cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$

$\Rightarrow \tan x/2 = t$ then, $2dt = \sec^2\frac{x}{2}\,dx$

- square term in deno. (degree 2)

eg: $\int \frac{1}{a+b\sin^2 x}$, $\frac{1}{a\sin^2 x+\cos^2 x}dx$

- divide num & deno by $\cos^2 x$ and put $\tan x = t \Rightarrow dt = \sec^2 x\,dx$.

Next, Use Standard integrals (I).

For integral of the form

$\int \frac{dx}{a\sin x \pm b\cos x}$ — $b \to \cos$ को coeff. $a \to \sin$ को coeff.

Putting $a = r\cos\theta$, $b = r\sin\theta$, $r = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$I = \int \frac{dx}{r\sin(x\pm\theta)} = \frac{1}{\sqrt{a^2+b^2}}\ln\tan\left(\frac{x}{2} \pm \frac{1}{2}\tan^{-1}\frac{b}{a}\right) + c$

(i) $\int a^x dx = \frac{a^x}{\log_e a} + c$ | # $\int e^{ax}dx = \frac{e^{ax}}{a} + c$

(ii) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + c = -\cos^{-1}x + c$

(iii) $\int \frac{dx}{1+x^2} = \tan^{-1}x + c = -\cot^{-1}x + c$

(iv) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x + c = -\operatorname{cosec}^{-1}x + c$

(i) $\int \tan x\,dx = \ln\sec x + c = -\ln\cos x + c$

(ii) $\int \cot x\,dx = \ln\sin x + c$

(iii) $\int \sec x\,dx = \log(\sec x + \tan x) + c = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$

(iv) $\int \operatorname{cosec} x\,dx = \log(\operatorname{cosec} x - \cot x) + c = \log(\tan x/2) + c$

Hyperbolic function

(i) $\int \sinh x\,dx = \cosh x + c$

(ii) $\int \cosh x\,dx = \sinh x + c$

(iii) $\int \tanh x\,dx = \log(\cosh x) + c$

(iv) $\int \coth x\,dx = \log(\sinh x) + c$

(v) $\int \operatorname{cosech} x\,dx = \log\tan h\left(\frac{x}{2}\right) + c$

(vi) $\int \operatorname{sech} x\,dx = 2\tan^{-1}\left(\tanh\frac{x}{2}\right) + c$

(vii) $\int \operatorname{sech}^2 x\,dx = \tanh x + c$

(viii) $\int \operatorname{cosech}^2 x\,dx = -\coth x + c$

(ix) $\int \operatorname{sech} x\tanh x\,dx = -\operatorname{sech} x + c$

(x) $\int \operatorname{cosech} x\coth x\,dx = -\operatorname{cosech} x + c$

★ $\sinh x = \frac{e^x - e^{-x}}{2}$; $\cosh x = \frac{e^x + e^{-x}}{2}$

(i) $\cosh^2 x - \sinh^2 x = 1$

(ii) $\cosh^2 x + \sinh^2 x = \cosh 2x$

(iii) $\sinh(-x) = -\sinh x$, $\cosh(-x) = \cosh x$

(iv) $\sinh 2x = 2\sinh x.\cosh x$

(v) $\cosh(x+y) = \cosh x.\cosh y + \sinh x.\sinh y$

(vi) $\sinh(x+y) = \sinh x\cosh y + \cosh x.\sinh y$

Derivative: $\sinh x$, $\cosh x$, $\tanh x$ → +ve

Other: $\operatorname{sech} x$, $\operatorname{cosech} x$, $\coth x$ → −ve

$\log_a x = \frac{\log x}{\log a}$

$\log_a x = \frac{1}{\log_x a}$

$\int \ln x\,dx = x\ln x - x + c.$

Standard integrals (I)

(a) $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$ ☆ {often confused}

(b) $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+c$

x^2ko अगाडिको sign

(c) $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+c$

(d) $\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c$ ☆ {often confused}

(e) $\int \frac{1}{\sqrt{x^2+a^2}}dx = \log(\underset{\text{sq. wala } x}{x}+\sqrt{x^2+a^2})+c = \sinh^{-1}\left(\frac{x}{a}\right)+c$

(f) $\int \frac{1}{\sqrt{x^2-a^2}}dx = \log(x+\sqrt{x^2-a^2})+c = \cosh^{-1}\left(\frac{x}{a}\right)+c$

Integrals reducible to standard forms. (when deno can't be factorized)

① (i) $\int \frac{dx}{ax^2+bx+c}$ (ii) $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ } only quadratic (not perfect square)

→ Make coeff. of x^2 unity by taking coeff. of x^2 outside the quadratic. Then integrand is converted to SI(I). {& make perfect sq. term inside}

② Linear & Quadratic

(i) $\int \frac{mx+e}{ax^2+bx+c}dx$ (ii) $\int \frac{mx+e}{\sqrt{ax^2+bx+c}}dx$

Put: Numerator $= p$(Derivative of quadratic) $+ q$

Integral of type $\int \frac{dx}{A\sqrt{B}}$ where A and B are linear or quadratic function of x.

(i) If $B \to$ linear, substitution; $B = z^2$

(ii) A linear & B quadratic, $A = 1/z$

(iii) A and B both quadratic, $x = 1/t$

Don't get confuse in a^2 & 1

Caution while simplification

+ & −

Standard integrals (II)

(a) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log(x+\sqrt{x^2+a^2})+c$

(b) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log(x+\sqrt{x^2-a^2})+c$

↳ a^2 agadiko sign

(c) $\int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c.$

For type $\int (mx+e)(\sqrt{ax^2+bx+c})dx$

Put: Linear function $= p$(Derivative of quadratic) $+ q$

$\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a\cos bx + b\sin bx)}{a^2+b^2}+c$

$\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2}+c$

$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+c = -\frac{1}{a}\operatorname{cosec}^{-1}\left(\frac{x}{a}\right)+c$

① $\int e^{ax}\left[f(x)+\frac{f'(x)}{a}\right] = \frac{e^{ax}f(x)}{a}+c$

↳ $\int e^{x}(f(x)+f'(x)) = e^{x}f(x)+c.$

(ii) $\int [xf'(x)+f(x)]dx = xf(x)+c$

$\frac{d}{dx}\{e^{ax}f(x)\} = ae^{ax}f(x) + e^{ax}f'(x)$

$\neq \cancel{e^{ax}[f(x)+f'(x)]}$

$\frac{d}{dx}(xf(x))+c = xf'(x)+f(x).$

① $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1}+c.\ \{n \neq -1\}$

② $\int \frac{f'(x)dx}{f(x)} = \ln f(x)+c\ \{n=-1\}$

(iii) $\int \frac{f'(x)dx}{\sqrt{f(x)}} = 2\sqrt{f(x)}+c$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R} = \frac{2\Delta}{abc}$

$\Delta \rightarrow$ Area of triangle
$2s = a+b+c \rightarrow$ perimeter.
$\Rightarrow s = \frac{a+b+c}{2}$

$\cos A = \frac{b^2+c^2-a^2}{2bc}$

iff, $\angle A = 60° \Rightarrow b^2+c^2-a^2 = bc$

→ Projection Formula: $\boxed{a = b\cos C + c\cos B}$
Used to find side of Δ when other two sides & their opp. angle are given.

$$\boxed{\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}}$$

- $\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C} \Rightarrow$ eq. triangle
- $\cos^2 A + \cos^2 B + \cos^2 C = 1 \Rightarrow$ Rt. ∠ed triangle

$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$
$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$

$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\Delta}{s(s-a)}$ ☆

$\because \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

→ $\Delta = \frac{1}{2}bc\sin A \Rightarrow \sin A = \frac{2\Delta}{bc}$

$\{A+B+C = \pi$ or $180°\}$

In any triangle, $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$

$\Delta = \frac{1}{2}\frac{a^2\sin B\sin C}{\sin(B+C)}$

→ $R \rightarrow$ Circumradius {Rad. of circumcircle of triangle}
$\frac{1}{2R} = \frac{2\Delta}{abc} \Rightarrow R = \frac{abc}{4\Delta}$

→ r: inradius [Rad. of incircle of triangle]

Dimension:
$R, r, r_1, r_2, s \rightarrow L$
$\sin A, \cos A \ldots \rightarrow 1$
$\Delta \rightarrow L^2$

- $r = \frac{\Delta}{s}$
- $r = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2}$
- $r = \frac{a\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}} = \frac{b\sin\frac{A}{2}\sin\frac{C}{2}}{\cos\frac{B}{2}} = \frac{c\sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{C}{2}}$

{In equilateral triangle, $r : R : r_1 = 1:2:3$}

- $r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

→ r_1, r_2, r_3: Radii of excribed circles of a triangle.

- $r_1 = \frac{\Delta}{s-a}$; $r_2 = \frac{\Delta}{s-b}$; $r_3 = \frac{\Delta}{s-c}$ ☆ {In equilateral Δ, $r_1 = r_2 = r_3$}
- $r_1 = s\tan\frac{A}{2}$
- $r_1 = 4R\sin\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\cos\frac{C}{2}$; $r_2 = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cdot\cos\frac{C}{2}$;
$r_3 = 4R\sin\frac{C}{2}\cdot\cos\frac{A}{2}\cdot\cos\frac{B}{2}$

{Assumption:
- equilateral triangle of side 2 unit
- Rt. angled triangle of side 3, 4, 5}

→ Principle solution(α)

$\sin\theta \to [-\frac{\pi}{2}, \frac{\pi}{2}]$; $\cos\theta \to [0, \pi]$; $\tan\theta \to (-\frac{\pi}{2}, \frac{\pi}{2})$

→ General Solution:-

I. $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$

$\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$

$\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$

II. $\sin^2\theta = \sin^2\alpha$, $\cos^2\theta = \cos^2\alpha$, $\tan^2\theta = \tan^2\alpha$ $\Big\} \Rightarrow \theta = n\pi \pm \alpha$

III. $\sin\theta = 0 = \sin 0 \Rightarrow \theta = n\pi$

$\cos\theta = 0 = \cos\frac{\pi}{2} \Rightarrow \theta = (2n+1)\frac{\pi}{2}$

$\tan\theta = 0 \Rightarrow \theta = n\pi$

IV. $\sin\theta = -\sin\alpha \Rightarrow \theta = n\pi + (-1)^n(-\alpha)$

$\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + (-\alpha)$

$\cos\theta = -\cos\alpha \Rightarrow \theta = 2n\pi + (\pi - \alpha)$

V. $\sin\theta = 1 \Rightarrow \theta = (4n+1)\frac{\pi}{2}$

$\sin\theta = -1 \Rightarrow \theta = (4n-1)\pi/2$

$\cos\theta = 1 \Rightarrow \theta = 2n\pi$

$\cos\theta = -1 \Rightarrow \theta = (2n+1)\pi$

→ Squaring while finding Roots ⇒ check the solutions to satisfy given equation.

→ For $a\cos\theta + b\sin\theta = c$
- Divide by $\sqrt{a^2+b^2}$

$|c| > \sqrt{a^2+b^2}$, no solution

$|c| \le \sqrt{a^2+b^2}$, has solution.

$$\boxed{\cosh x = \frac{e^x + e^{-x}}{2} \qquad \sinh x = \frac{e^x - e^{-x}}{2}}$$

→ Given two variables & asked to find P.V & General value then.

Principle value (PV) of θ : b/w 0 & 2π.

If x lies in

1st quad, $PV = x$

2nd quad, $PV = \pi - x$

3rd quad, $PV = \pi + x$

4th quad, $PV = 2\pi - x$

$\Big\}$ & General value $= 2n\pi + PV$.

→ Convert to sin & cos if question is complex.

→ $\sin^{-1}x$ → Angle ; $x = \sin\theta$ → Real number.

ITF

$f(x)$	Domain (x)	Range ($f(x)$) (PV)
$\sin^{-1}x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1}x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\tan^{-1}x$	$(-\infty, \infty)$ R	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$\operatorname{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$
$\sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \{\frac{\pi}{2}\}$
$\cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$

→ $\sin^{-1}(-x) = -\sin^{-1}x$; $\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\tan^{-1}(-x) = -\tan^{-1}x$; $\sec^{-1}(-x) = \pi - \sec^{-1}x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$; $\cot^{-1}(-x) = \pi - \cot^{-1}x$.

→ $\sin^{-1}x + \cos^{-1}x = \pi/2$

$\tan^{-1}x + \cot^{-1}x = \pi/2$

$\sec^{-1}x + \operatorname{cosec}^{-1}x = \pi/2$.

→ $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$

$2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

→ Assumption : suitable value belonging to domain.

→ 'HAT' method: Using options to check answer.

→ Cast Rule

Quadrant → '+' or '−'

→ $\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

→ $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$

→ $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

(sine or cosine ko formula ma muni ~~tan~~ $\sec^2$ aauna paryo) {∵ $1+\tan^2 A = \sec^2 A$}

→ $\sin 3A = 3\sin A - 4\sin^3 A$

→ $\cos 3A = 4\cos^3 A - 3\cos A$

→ $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$ { $\dfrac{\text{sum of one at a time} - \text{sum of three at a time}}{1 - \{\text{sum of two at a time}\}}$

→ $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

→ $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

→ $\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{B-A}{2}\right)$

☆ Assuming & Getting values

☆ Differentiation

→ $\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B$

$\cos(A+B)\cdot\cos(A-B) = \cos^2 A - \sin^2 B$

☆ $\boxed{\sin\alpha \le 1}$

↑ If difficult to observe anything in question use this

→ If $A+B+C = \pi$ (or $180°$)

- $\sin 2A + \sin 2B + \sin 2C = 4\sin A\cdot\sin B\sin C$
- $\cos 2A + \cos 2B + \cos 2C = 1 - 4\cos A\cos B\cos C$
- $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\cos\frac{C}{2}$
- $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Reverse $(-)\to(+)$, $\sin \to \cos A$

- $\tan A + \tan B + \tan C = \tan A\tan B\tan C$
- $\cot A\cot B + \cot B\cot C + \cot C\cot A = 1$.

- If $\sec\theta \pm \tan\theta = a$, $\sec\theta \mp \tan\theta = 1/a$.
- If $\operatorname{cosec}\theta \pm \cot\theta = a$, $\operatorname{cosec}\theta \mp \cot\theta = 1/a$.

⇓ $\sec^2\theta - \tan^2\theta = 1$

→ Domain and Range of T(f)

f(θ)	(θ) Domain	Range f(θ)	f(θ)	Domain	Range
$\sin\theta$	R i.e. $(-\infty,\infty)$	$[-1,1]$	$\cos\theta$	R	$[-1,1]$
$\tan\theta$	$R - (2n+1)\frac{\pi}{2}$	R $(-\infty,\infty)$	$\sec\theta$	$R-(2n+1)\frac{\pi}{2}$	$(-\infty,-1] \cup [1,\infty)$
$\operatorname{cosec}\theta$	$R - n\pi$	$(-\infty,-1]\cup[1,\infty]$	$\cot\theta$	$R - n\pi$	R

→ • $c+\sqrt{a^2+b^2} \ge a\sin\theta \pm b\cos\theta + c \ge c - \sqrt{a^2+b^2}$

- $\sin^2\theta + \operatorname{cosec}^2\theta \ge 2$
- $\cos^2\theta + \sec^2\theta \ge 2$
- $\tan^2\theta + \cot^2\theta \ge 2$.

• If $z = ax + by$ such that $xy = 1$, then min. value of z is obtained as A.M. ≥ G.M.

Complex Number

→ $z = a + ib = (a, b)$ { x-axis → real axis, y-axis → im-axis }

→ Every real number is complex number (with im part = 0)

→ If $z = (a, b)$ & $w = (c, d)$, Then

* $z = w$ iff $a = c$ and $b = d$.
* $z + w = (a + c, b + d)$ ↳ rep. as diagonal of ||gm
* $z - w = (a - c, b - d)$
* $z \cdot w = (a, b) \cdot (c, d)$

  $z \cdot w = (ac - bd, ad + bc)$
* $\frac{z}{w} = \frac{(a+ib)(c-id)}{(c+id)(c-id)} = \left(\frac{ac+bd}{c^2+d^2}\right) + i\left(\frac{bc-ad}{c^2+d^2}\right)$

→ Additive inverse of $z = (a, b)$ is $-z$ i.e. $-z = (-a, -b)$

→ Aditive identity is $(0, 0)$ } For all complex number.

→ Multiplicative identity is $(1, 0)$ }

→ Multiplicative inverse of $z = (a, b)$ is $1/z$.

$$\frac{1}{z} = \frac{1}{a+ib} = \frac{a-ib}{a^2+b^2} = \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$$

→ Conjugate of $z = a + ib$ is $\bar{z}$ i.e. $\bar{z} = a - ib$.

→ $z = a + ib$ { → Purely real if $b = 0$ → $\boxed{z = \bar{z}}$; → Purely imaginary if $a = 0$ → $\boxed{z = -\bar{z}}$ } ✓

→ i: imaginary unit of complex number

- $i = (0, 1)$
- $i^2 = (0, 1) \cdot (0, 1) = (0, -1) = -1$
- $i^{4n} = 1$ } where n is an integer
- $i^{4n+2} = -1$
- $\frac{1}{i} = \frac{1}{i} \times \frac{i}{i} = -i$
- Sum of any four consecutive powers of i is zero.

→ Two complex numbers follows associative & distributive law of addition & multiplication and commutative law of addition ~~but not of multiplication~~

→ Properties of complex conjugate:

(i) $\overline{(\bar{z})} = z$

(ii) $z + \bar{z} = 2\,\mathrm{Re}(z)$

(iii) $z - \bar{z} = 2\,\mathrm{im}(z)$

(iv) $\overline{z + w} = \bar{z} + \bar{w}$

(v) $\overline{zw} = \bar{z} \cdot \bar{w}$

(vi) $\overline{z_1 z_2 z_3} = \bar{z}_1 \cdot \bar{z}_2 \cdot \bar{z}_3$

(vii) $\overline{z_1 + z_2 + z_3} = \bar{z}_1 + \bar{z}_2 + \bar{z}_3$

(viii) $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

☆ conjugate gets distributed to each complex number.

Modulus of complex number : $z = a+ib$

$\hookrightarrow |z| = \sqrt{a^2+b^2}$

- $|z_1| \rightarrow$ distance of z_1 from origin.
- $|z_2 - z_1| \rightarrow$ dist. b/w z_1 & z_2 i.e. dist. of z_2 from z_1

(i) $|z| = |\bar{z}| = |-z|$

(ii) $|z| = 0$ iff $z = 0$

(iii) $-|z_1| \le Re(z_1) \le |z_1|$

(iv) $-|z_1| \le Im(z_1) \le |z_1|$

(v) $|z_1 z_2| = |z_1| \cdot |z_2|$

(vi) $|z^n| = |z|^n$

(vii) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ $(z_2 \neq 0)$

$\boxed{(i)^i \rightarrow \text{Real no} \rightarrow e^{-\pi/2}}$

(viii) $|z+w| \le |z| + |w|$ } Triangle law of inequality

(ix) $|z-w| \ge |z| - |w|$

(x) $|z|^2 = z \cdot \bar{z}$

(xi) $|z| = a \Rightarrow \bar{z} = a^2/z$

(xii) $|z-w| \le |z| + |w|$

(xiii) $|z_1+z_2|^2 + |z_1-z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

(xix) $|z_1+z_2| = |z_1-z_2| \Leftrightarrow arg(z_1) - arg(z_2) = \pi/2$

(x) $|z_1+z_2| = |z_1| + |z_2| \Leftrightarrow arg(z_1) = arg(z_2)$

Polar Form of complex number

If $\boxed{z = x+iy}$ then, polar form of z is

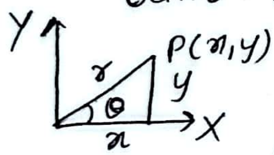
Cartesian form.

$* \; z = r\underbrace{(\cos\theta + i\sin\theta)}_{\rightarrow \; Cis\theta}$

$\boxed{r = \sqrt{x^2+y^2}}$

$r = \sqrt{x^2+y^2}$

$\tan\theta = y/x \Rightarrow \theta = \tan^{-1}(y/x)$ } $\theta \rightarrow$ argument or amplitude.



Euler form

$z = x+iy$

$\hookrightarrow z = re^{i\theta}$ $\hookrightarrow Cis\theta$

→ used to find big/bad power of z.

$e^{i\theta} = \cos\theta + i\sin\theta$

Table for arguments (θ)

- $\theta \in [-\pi, \pi]$
- Argument of $z=0$ is not defined.

Quadrants	1st	2nd	3rd
Argument	θ	$\pi-\theta$	$-\pi+\theta$

$\boxed{4th \rightarrow -\theta}$

	Complex number		Value of argument
(i)	$z = x, x>0$	purely real	0
(ii)	$z = -x, x<0$		$-\pi$
(iii)	$z = iy, y>0$	purely im.	$\pi/2$
(iv)	$z = -iy, y<0$		$-\pi/2$ or $3\pi/2$
(v)	$z_1 z_2$		$arg(z_1) + arg(z_2)$
(vi)	$\frac{z_1}{z_2}$		$arg(z_1) - arg(z_2)$
(vii)	z^n		$n \, arg(z)$
(viii)	$\bar{z}$		$2\pi - arg(z)$

For $z = x+iy$

$\theta = \tan^{-1}(y/x)$

Let, $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

Then,

$* \; z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$

$* \; \frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)]$

De-Moivre's Theorem

For any integer n (positive or negative or zero),

$\boxed{[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)}$

& the n distinct roots are given by:

$z_K = \sqrt[n]{r}(\cos\theta_K + i\sin\theta_K)$

where, $\theta_K = \frac{\theta + K\,360°}{n}$ & $K = 0, 1, 2, \ldots (n-1)$

Square roots of a complex number; $z = a+ib$

- Let, $x+iy = \sqrt{a+ib}$

$(x+iy)^2 = a+ib$

then, $x^2 = \frac{\sqrt{a^2+b^2}+a}{2}$ and $y^2 = \frac{\sqrt{a^2+b^2}-a}{2}$

- xy and b must have same sign

↳ $b = -ve$; take x and y of alternate sign (opposite signs)

$b = +ve$; take x and y of same sign.

Cube roots of unity

$Z = \sqrt[3]{1}$

$Z^3 - 1 = 0 \Rightarrow (Z-1)(Z^2+Z+1) = 0$

$Z = 1$, $Z = \frac{-1 \pm \sqrt{3}i}{2}$.

$1, \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$ i.e. $1, \omega, \omega^2$ } $\omega^2 = \bar{\omega}$

$Z^2 - Z + 1 = 0 \rightarrow$ Roots $-\omega$ & $-\omega^2$

$Z^2 + Z + 1 = 0 \rightarrow$ Roots ω & ω^2

Properties of cube roots of unity

- $\boxed{1+\omega+\omega^2 = 0} \Rightarrow \omega+\omega^2 = -1$
- $\omega \cdot \omega^2 = \omega^3 = 1$ } Product of imaginary roots $= 1$.
- $\boxed{\omega^{3n} = 1}$
- cube roots of unity forms an eq. triangle in argand plane
- Each im. cube root of unity is square of other
  i.e. $(\omega)^2 = \omega^2$ & $(\omega^2)^2 = \omega$.

$\omega^{3K} + \omega^{3K+1} + \omega^{3K+2} = 0$, $K \in Z$.

Fourth roots of unity $\rightarrow \pm 1, \pm i$

n^{th} root of unity

↳ forms a regular polygon having n sides {eq. Δ, sq. . .}

↳ sum of n roots $= 0$ i.e. $1+\omega+\omega^2+\ldots+\omega^{n-1} = 0$ } $(n>1)$

↳ Product $\rightarrow (-1)^{n-1}$

Area of Δ formed by z, iz & $z+iz$ as vertices $= \frac{1}{2}|z|^2$

Types of Relation

(i) Reflexive Relation : $[(a,a) \in R$, for every $a \in A] \rightarrow$ Reflexive Relation R in set A

eg: $A = \{1,2,3\}$; $R = \{(1,1),(2,2),(3,3),(1,3)\} \rightarrow$ Reflexive Relation.

(ii) Symmetric Relation: $[(x,y) \in R \Rightarrow (y,x) \in R]$

eg: $A = \{1,4,5\}$; $R = \{(4,5),(5,4),(1,1),(1,4),(4,1)\}$

(iii) Antisymmetric relation : $[(x,y) \in R$ and $(y,x) \in R \Rightarrow x = y]$

(iv) Transitive Relation : $[(x,y) \in R$ and $(y,z) \in R \Rightarrow (x,z) \in R]$

eg: $A = \{2,4,8\}$ $R = \{(2,4),(4,8),(2,8)\}$ $\because 2<4$ and $4<8 \Rightarrow 2<8$

(v) Equivalence Relation : Reflexive, symmetric & transitive

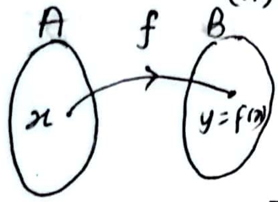
$R \subset A \times B$.

Function

~~From~~ Relation from set A to set B is said to be function if

(i) Every element of set A associates

(ii) TO a unique (single) element of set B.



$A \rightarrow$ Domain $\rightarrow x$ which satisfies given f(x)

$B \rightarrow$ Codomain

$y = f(x) \in B \rightarrow$ Range { Image in B of set A }

↳ { values of y for all possible values of x }

$\rightarrow$ Range $\subset B$

Inequility change (reverse) $\rightarrow$ (i) Mutiplied by '-1'
(ii) Reciprocal

Modulus function $\rightarrow \boxed{|\cdot| \geq 0}$

$$y = |x| \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$

$D_f = R = (-\infty, \infty)$

$R_f = [0, \infty)$

$= R - (-\infty, 0)$

Equal function : $f(x) = g(x)$ {for all $x \in A$}

(i) Domain of f = Domain of g $\Rightarrow D_f = D_g$.

(ii) $f(x) = g(x)$ for all $x \in D_f$.

Number of functions from finite set A into a finite set B $\longrightarrow \{n(B)\}^{n(A)}$ { [TO] From

Types of function:-

(i) One to one or injection

↳ If diff. elements in A have diff. images in B.

i.e. For $x_1, x_2 \in A$

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ ie

or $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ ie

Number of function from a finite set A into a finite set B

↳ $[n(B)]^{n(A)}$

(ii) Onto or Surjective function.

↳ Every element of B has at least one pre-image in A, otherwise it is said to be into.

i.e. $f(A) = B \rightarrow$ onto functions {B की हर element has pre-image in A}

$f(A) \subset B \rightarrow$ into function.

Every polynomial function $f: R \rightarrow R$ of degree odd is onto & constant function with singleton co domain → onto.

→ eg: $f(x) = 3x+2 \Rightarrow x = \frac{y-2}{3} \in Q \quad f: Q \rightarrow Q$ {Q - Rational}

$f\left(\frac{y-2}{3}\right) = 3\left(\frac{y-2}{3}\right) + 2 = y \Rightarrow y$ is image of x i.e. $\left(\frac{y-2}{3}\right)$

$\Rightarrow f$ is onto function

(iii) Bijective function.

↳ Both one to one and onto i.e. Both injective & surjective

• Constant function, $f(x) = c$ {singleton}

• Identity function, $y = f(x) = x$

• Linear function, $y = f(x) = mx + c$ {$m, c \rightarrow$ constant & $m \neq 0$}

Exponential function, $y = f(x) = a^x, \; x \in R$

$\Downarrow$

$\underline{a > 0 \;\&\; a \neq 1}$.

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$
$$= \lim_{h \to 0} (1+h)^{1/h}$$

• Polynomial function, $y = f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n$

$a_0 \neq 0$ & n is non-negative integer.

Polynomial function of degree n

• Logarithmic function: inverse of exponential

If $y = a^x, \; a \neq 1 \;\&\; a > 0$ its inverse is

$x = a^y$

$\Rightarrow \boxed{y = \log_a x}$

• Reciprocal function: $y = f(x) = \frac{1}{x}, \; x \neq 0$

$D_f = R - \{0\}$

$R_f = R - \{0\}$

$xy = 1 \rightarrow$ Rect. hyperbola.

(iv) Inverse function,

If $f: A \rightarrow B$; then $f^{-1}: B \rightarrow A$ is inverse function of f

Note ① For inverse function to exist; the function must be bijective {i.e. one to one & onto}

② If A and B are inverses of each other, then

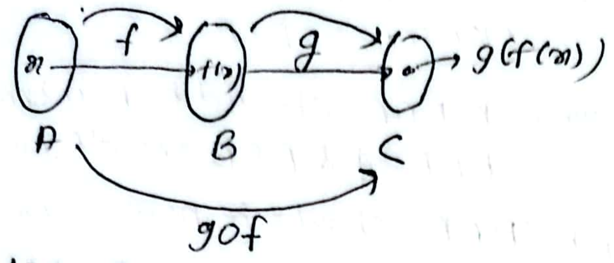
• Domain of A = Range of B
• Range of A = Domain of B

} can be used as trick as well.

Composite function

let; $f: A \to B$ and $g: B \to C$ then composite function of f and g denoted by gof is a function from A to C.

i.e. $gof: A \to C$, defined by $(gof)(x) = g\{f(x)\}$ for all $x \in A$



Even function
↳ $f(-x) = f(x)$

Odd function
↳ $f(-x) = -f(x)$

Periodic function

Period

(i) $\sin^n x, \cos^n x, \sec^n x, \operatorname{cosec}^n x \longrightarrow 2\pi \to n$ is odd
$\pi \to n$ is even

(ii) $\tan^n x, \cot^n x \longrightarrow \pi \to n$ is even or odd.

(iii) $|\sin x|, |\cos x|, |\tan x|, |\sec x|, |\operatorname{cosec} x|, |\cot x| \longrightarrow \pi$

(iv) $|\sin x| + |\cos x|$, $|\tan x| + |\cot x|$, $|\sec x| + |\operatorname{cosec} x| \to \pi/2$

(v) $f(x)$ is periodic with period $\to T$, then $f(ax+b)$ is periodic with period $\frac{T}{|a|}$

(vi) If $f(x), g(x), h(x)$ are periodic with period T_1, T_2, T_3 then period of $af(x) \pm b g(x) \pm c h(x)$ is
↳ L.C.M. of $\{T_1, T_2, T_3\}$ {where. a, b, c are constant}

Note:

$$\text{LCM of } \frac{a}{a_1}, \frac{b}{b_1}, \frac{c}{c_1} = \frac{\text{LCM of } \{a, b, c\}}{\text{H.C.F. of } \{a_1, b_1, c_1\}}$$

Period → can be checked from options as well

$A \times B \to$ cartesian product

If A contains m elements and B contains n elements then number of elements in $A \times B$ is mn

No. of possible relation from set A to set B $= 2^{mn}$ } Number of subsets.

No. of functions from finite set A to finite set B $= [n(B)]^{n(A)}$

If $o(A \cap B) = n$, then $o((A \times B) \cap (B \times A)) = n^2$ }

$|x| < a \Rightarrow -a < x < a$

$\phi \to$ empty set (finite set) } $\{0\} \to$ set containing only one element 0 i.e. it is singleton set

Power set of A: set of all subsets of A if $n(A) = n$.

↳ No. of ~~subsets~~ subset in $P(A) = 2^n$

↳ No. of proper subsets $= 2^n - 1$ {except A}

↳ No. of non-empty proper subset $= 2^n - 2$ {except A & ϕ}

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Proper subset: If A is proper subset of B if $A \subset B$ and $A \neq B$

Equivalent set → same no. of elements.

$A \Delta B = (A-B) \cup (B-A)$
$= (A \cup B) - (A \cap B)$ } set of non-common elements of A & B.

For $n(A \cup B)$ to be maximum, $n(A \cap B)$ should be minimum
$\boxed{n(A \cup B) \leq n(U)}$ i.e. $n(A \cap B) = 0$ i.e. $A \cap B = \phi$ & A & B are disjoint set.

For $n(A \cup B)$ to be min, $n(A \cap B)$ should be maximum
i.e. $A \subset B$.

Idempotent law: $A \cup A = A$ & $A \cap A = A$

Identity law: $A \cup \phi = A$; $A \cap \phi = \phi$; $A \cup U = U$; $A \cap U = A$

De Morgan's law: $\overline{(A \cup B)} = (\overline{A} \cap \overline{B})$ & $\overline{(A \cap B)} = (\overline{A} \cup \overline{B})$

$A-(B \cap C) = (A-B) \cup (A-C)$
$A-(B \cup C) = (A-B) \cap (A-C)$
$A \cap (B-C) = (A \cap B) - (A \cap C)$

$x \notin B \cup C$ = $x \notin B$ and $x \notin C$
$x \notin B \cap C$ = $x \notin B$ or $x \notin C$

$|x+y| \leq |x| + |y|$; $|x-y| \geq |x| - |y|$; $|x|^2 = x^2$

$A \times B \neq B \times A$; $A \subseteq B$ then $A \times C \subseteq B \times C$ for any set C.

Relation on A ⇒ Relation from A to A.

If $n(A) = n$; Number of relation on A $= 2^{n \cdot n} = 2^{n^2}$

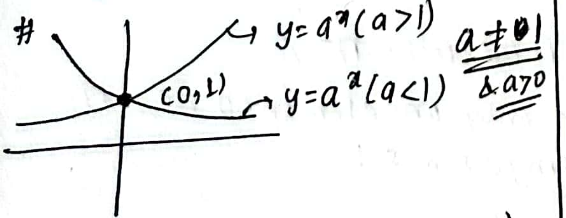
Relation $\subset (A \times B)$
$R = \{(x,y): x \in A, y \in B\} \subseteq A \times B$
$R^{-1} = \{(y,x): y \in B, x \in A\} \subseteq B \times A$

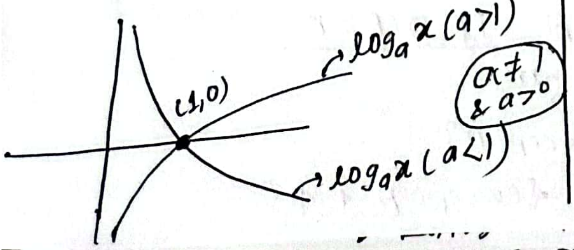
$(2^x)^2 = 2^{2x}$; 2^{x^2} if $2^{2x} = 2^{x^2}$ ⇒ $2x = x^2$ or, $x^2 - 2x = 0$

$\sqrt{x^2} = |x|$ or $\pm x$.

$y = e^x ⇒ D_f = R$; $R_f (0, \infty)$ } $y = a^x ⇒ D_f = R$; $R_f = (0, \infty)$, $a \neq 1$

#





$\log_a a = 1$, $\log_a 1 = 0$ $[a > 0, a \neq 1]$
$\log_a b = \frac{1}{\log_b a}$ [base > 0 & ≠ 1]
$\log_a b = \log_c b \cdot \log_a c = \frac{\log_c b}{\log_c a}$

$\log_a (xy) = \log_a x + \log_a y$
$\log_a (x/y) = \log_a x - \log_a y$
$\log_{a^n} (x) = \frac{1}{n} \log_a x$
$\log_{a^n} (x^m) = \frac{m}{n} \log_a x$
} $x > 0$, $y > 0$, base > 0 & ≠ 1

Binomial Theorem

$n! = n \times (n-1)!$

$(x+y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1}y^1 + \dots {}^nC_r x^{n-r}y^r + \dots {}^nC_n y^n$

$n \to$ +ve integer value (Natural number) $\left\{ \begin{matrix} 0! = 1 \\ 1! = 1 \end{matrix} \right\}$

${}^nC_r \to$ Binomial coefficient : ${}^nC_r = \dfrac{n!}{(n-r)!\,r!}$

☆ ${}^5C_2 = \dfrac{5\times4}{2\times1}$; ${}^6C_3 = \dfrac{6\times5\times4}{3!}$ } upto 3 numbers (lower index) } → factorial of lower index.

- $\boxed{{}^nC_r = {}^nC_{n-r}}$ $\boxed{{}^nC_1 = n}$ $\boxed{{}^nC_n = 1}$, $\boxed{{}^nC_0 = 1}$
- General term of $(x+y)^n$

$\boxed{T_{r+1} = {}^nC_r\, x^{n-r} y^r}$ $\left\{ \begin{matrix} r \to \text{integer} ; r \neq \text{negative \& fraction} \\ r \leq n \end{matrix} \right\}$

$r \to (r+1)^{th}$ term eg: $r = 5 \to 6^{th}$ term
$r = 4 \to 5^{th}$ term

- In expanpasion of $(x+y)^n \to (n+1)$ term
- p^{th} term from end $\Rightarrow (n-p+2)^{th}$ term from Beginning
- If coefficient of T_r, T_{r+1}, T_{r+2} ~~are~~ of expansion $(1+x)^n$ are in A-P then $\boxed{(n-2r)^2 = n+2}$
- In G.P. : $S_n = \dfrac{a(r^n - 1)}{r-1}$ } n = number of terms

☆
(i) $(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots + \infty$
(ii) $(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots + \infty$
For $|x| < 1$
(iii) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$
(iv) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots$
(v) $(1-x)^{-1/2} = 1 + \frac{1}{2}x + \frac{1\cdot3}{2\cdot4}x^2 + \dots$

- Middle term: $(x+y)^n$

→ $\boxed{n \to \text{even}}$ → odd terms → only 1 middle term

Middle term: $t_{\left(\frac{n}{2}+1\right)}$

→ $\boxed{n \to \text{odd}}$ → even terms → Two middle terms

Middle terms: $T_{\left(\frac{n+1}{2}\right)}$, $T_{\left(\frac{n+1}{2}+1\right)}$

(Q) Coefficient of x^5 in the expansion of $(1+x)^3(1-x^2)^7$

(a) ${}^7C_5 - {}^5C_3$ (b) ${}^{14}C_5$ (c) ${}^3C_1\,{}^7C_2 - {}^3C_3\,{}^7C_1$ (d) ${}^7C_5\,{}^5C_1 - {}^7C_2\,{}^5C_3$

Greatest Binomial coefficient, Numerically Greatest Coefficient & Numerically greatest term (for given value of variable)

→ Greatest Binomial coefficient ⇒ Binomial coeff. of middle term/s.

- $(1+x)^n$ → Greatest Binomial coefficient / Numerically Greatest term } same.

$(x+y)^n$: $\boxed{m = \dfrac{n+1}{1+\left|\frac{x}{y}\right|}}$ → Numerically greatest coefficient ($x = y = 1$)

n → index $\boxed{m-1 \le r \le m}$ { if m → decimal, r → integral value b/w decimal values }

m → Position of Numerically Greatest ~~term~~ coefficient

No. of terms in the expansion

- $(x+y)^n$ ⟶ $(n+1)$ term

$(x+y+z)^n$ ⟶ $\boxed{^{n+r-1}C_{r-1}}$ { r = no. of variables, here: $x, y, z \Rightarrow r = 3$

Polynomial in single variable having same sign

Eg: $(4x^3 + 5x^2 + x + \underset{x^0}{1})^6$ { variable → x, sign → + }

$$\boxed{\text{No. of terms} = N = \left(\text{Higher degree} - \text{Lower degree}\right) \times \text{Index} + 1}$$

$N = (3-0) \times 6 + 1 = \underline{19 \text{ terms}}$.

Sum of coefficients : Put variables = 1.

Eg: sum of coeff. in exp. $(ax+by)^n$

sum of coeff = $(a+b)^n$ } For $x = y = 1$.

$\boxed{(x+y)^n = \sum_{r=0}^{n} {^nC_r}\, x^{n-r} y^r}$

$(1+x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \dots + C_n x^n$

$(1-x)^n = C_0 - C_1 x + C_2 x^2 - C_3 x^3 + \dots + (-1)^n C_n x^n$

Sum of binomial coefficients: $\boxed{C_0 + C_1 + C_2 + C_3 + \dots + C_n = 2^n}$

sum of even & odd coefficients: $\boxed{C_0 + C_2 + C_4 + C_6 + \dots = C_1 + C_3 + C_5 + \dots = 2^{n-1}}$

$(1+x)^n$

when, n is not natural number & n is fractional or negative

& $\boxed{|x|<1}$ ↳ coeff. can't be expressed as ${}^nC_0, {}^nC_1, \ldots$

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \infty \text{ terms}$$

$|x| < a \Rightarrow -a < x < a$

Q) Coefficient of x^3 in expansion $\frac{(1+3x)^2}{1-2x}$

(i) 50 (ii) 32 (iii) 48 (iv) 55

Sum of Product of Binomial coefficients $\{C_0C_r + C_1C_{r+1} + \ldots\}$

If difference of lower index is constant

$$= \frac{(2n)!}{(n-r)!\,(n+r)!}$$

• If difference of lower indices
• $r \to$ difference of lower indices

eg: $C_0C_2 + C_1C_3 + C_2C_4 + \ldots + C_{n-2}C_n$

$$= \frac{(2n)!}{(n-2)!\,(n+2)!}$$

☆ Putting value of n to find sum

↓

If given other constant in sum like a, b... then take $n \geq 2$

$a_0C_0 + a_1C_1 + a_2C_2 + a_3C_3 + \ldots + a_nC_n$

If a_0, a_1, a_2, a_3 are in A.P, then

$= (2a_0 + nd)2^{n-1}$ } $a_0 \to$ first term, $d \to$ common difference.

or $S = (a_0 + a_n)\,2^{n-1}$

$(x+a)^n + (x-a)^n$

Total number of terms $= \left(\frac{n}{2}+1\right)$ {if n is even

(After Simplification) & $\left(\frac{n+1}{2}\right)$ if n is odd

$(x+a)^n - (x-a)^n$

Total number of terms in the expansion after simplification

$= \frac{n}{2}$ if n is even

$= \frac{n+1}{2}$ if n is odd

In expansion $\left(x^p + \frac{1}{x^q}\right)^n$

For term independent of x, } $r = \frac{n \times p}{p+q}$

Ans: $C(n, r)$

$r \to$ -ve or fraction

No term independent of x.

→ For $\left(x^p - \frac{1}{x^q}\right)^n$: Independent term of x

r can be found in same way for other question of x^p

$\left\{ r = \frac{n \times p}{p+q} \right\}$ Ans: $C(n, r)$ if r is even
$-C(n, r)$ if r is odd.

But be cautious while finding coeff.

(Q) The independent term in expansion

$$\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - \sqrt{x}}\right)^{10} \text{ is}$$

$\frac{1}{(-ve)!} \Rightarrow$ Considered 0.

(a) $C(10, 0)$ (b) $C(10, 3)$ (c) $C(10, 10)$ (d) $C(10, 4)$

$\left\{ (-ve)! = \text{not defined} \right\}$

$\left\{ e^x = \lim_{n \to \infty}\left(1 + \frac{1}{n}\right)^{nx} = \lim_{n \to 0}(1+n)^{x/n} \right\}$

For $\left(a(x)^p \pm \frac{b}{x^q}\right)$

To find coefficient of x^m in expansion

- $r = \frac{n \times p - m}{p+q}$

$\boxed{\text{In AP } S_n = \frac{n}{2}[2a + (n-1)d]}$

No terms in 'n' innumerator for finding t_n.

- Next, use above r to find coefficient.

• $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ $\{e^{a+b} = e^a \cdot e^b\}$

- $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ $\{\text{For } -1 < x \le 1\}$
- $\left(a^x = 1 + \frac{x}{1!}\log_e a + \frac{x^2}{2!}(\log_e a)^2 + \cdots\right) \mapsto \underline{a > 0}$ $\{$factorial → ! ⇒ In exp. of $e^x\}$
- $\log_e(1-x) = -\frac{x}{1} - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} \cdots$ $\{\text{For } -1 \le x < 1\}$

The coefficient of x^n in the expansion $\log(1+x+x^2)$ is

- $-2/n$ if n is multiple of 3
- $1/n$ if n is not multiple of 3

$\log a + \log b = \log(ab)$

$\log a - \log b = \log\left(\frac{a}{b}\right)$

In expansion $(x+1)(x+2)(x+3) \cdots (x+n)$
coeff. of $x^{n-1} = \frac{n(n+1)}{2}$

$\boxed{\log_e\left(\frac{1+x}{1-x}\right) = 2\left(\frac{x}{1} + \frac{x^3}{3} + \frac{x^5}{5} + \cdots \text{ to } \infty\right) \text{ for } -1 < x < 1 \text{ i.e. } |x| < 1.}$

If the coeff. of p^{th} and q^{th} terms in the expansion of $(1+x)^n$ are equal, then $\boxed{p + q = n + 2}$

$\boxed{\text{In GP } S_n = \frac{a(r^n - 1)}{r - 1}}$

$\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots \infty$ $\}$ only even terms
(don't forget !!)

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